VISIT…

ELEMENTARY CALCULUS

An Infinitesimal Approach

Second Edition

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Dedicated to my sons, Randall, Jeffrey, and Thomas

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PREFACE TO THE SECOND EDITION

In this second edition, many changes have been made based on nine years of classroom experience. There are major revisions to the first six chapters and the Epilogue, and there is one completely new chapter, Chapter 14, on differential equations. In addition, the original Chapters 11 and 12 have been repackaged as three chapters: Chapter 11 on partial differentiation, Chapter 12 on multiple integration, and Chapter 13 on vector calculus.

Chapter 1 has been shortened, and much of the theoretical material from the first edition has been moved to the Epilogue. The calculus of transcendental functions has been fully integrated into the course beginning in Chapter 2 on derivatives. Chapter 3 focuses on applications of the derivative. The material on setting up word problems and on related rates has been moved from the first two chapters to the beginning of Chapter 3. The theoretical results on continuous functions, including the Intermediate, Extreme, and Mean Value Theorems, have been collected in a single section at the end of Chapter 3. The development of the integral in Chapter 4 has been streamlined. The Trapezoidal Rule has been moved from Chapter 5 to Chapter 4, and a discussion of Simpson's Rule has been added. The section on area between two curves has been moved from Chapter 6 to Chapter 4. Chapter 5 deals with limits, approximations, and analytic geometry. An extensive treatment of conic sections and a section on Newton's method have been added. Chapter 6 begins with new material on finding a volume by integrating areas of cross sections.

Only minor changes and corrections have been made to Chapters 7 through 13. The new Chapter 14 gives a first introduction to differential equations, with emphasis on solving first and second order linear differential equations. In Section 14.4, infinitesimals are used to give a simple proof that every differential equation $y' = f(t,y)$, where f is continuous, has a solution. The proof of this fact is beyond the scope of a traditional elementary calculus course, but is within reach with infinitesimals.

I wish to thank all my friends and colleagues who have suggested corrections and improvements to the first edition of the book.

H. Jerome Keisler

PREFACE TO THE FIRST EDITION

The calculus was originally developed using the intuitive concept of an infinitesimal, or an infinitely small number. But for the past one hundred years infinitesimals have been banished from the calculus course for reasons of mathematical rigor. Students have had to learn the subject without the original intuition. This calculus book is based on the work of Abraham Robinson, who in 1960 found a way to make infinitesimals rigorous. While the traditional course begins with the difficult limit concept, this course begins with the more easily understood infinitesimals. It is aimed at the average beginning calculus student and covers the usual three or four semester sequence.

The infinitesimal approach has three important advantages for the student. First, it is closer to the intuition which originally led to the calculus. Second, the central concepts of derivative and integral become easier for the student to understand and use. Third, it teaches both the infinitesimal and traditional approaches, giving the student an extra tool which may become increasingly important in the future.

Before describing this book, I would like to put Robinson's work in historical perspective. In the 1670's, Leibniz and Newton developed the calculus based on the intuitive notion of infinitesimals. Infinitesimals were used for another two hundred years, until the first rigorous treatment of the calculus was perfected by Weierstrass in the 1870's. The standard calculus course of today is still based on the "ε, δ definition" of limit given by Weierstrass. In 1960 Robinson solved a three hundred year old problem by giving a precise treatment of the calculus using infinitesimals. Robinson's achievement will probably rank as one of the major mathematical advances of the twentieth century.

Recently, infinitesimals have had exciting applications outside mathematics, notably in the fields of economics and physics. Since it is quite natural to use infinitesimals in modelling physical and social processes, such applications seem certain to grow in variety and importance. This is a unique opportunity to find new uses for mathematics, but at present few people are prepared by training to take advantage of this opportunity.

Because the approach to calculus is new, some instructors may need additional background material. An instructor's volume, "Foundations of Infinitesimal

Calculus," gives the necessary background and develops the theory in detail. The instructor's volume is keyed to this book but is self-contained and is intended for the general mathematical public.

This book contains all the ordinary calculus topics, including the traditional limit definition, plus one extra tool—the infinitesimals. Thus the student will be prepared for more advanced courses as they are now taught. In Chapters 1 through 4 the basic concepts of derivative, continuity, and integral are developed quickly using infinitesimals. The traditional limit concept is put off until Chapter 5, where it is motivated by approximation problems. The later chapters develop transcendental functions, series, vectors, partial derivatives, and multiple integrals. The theory differs from the traditional course, but the notation and methods for solving practical problems are the same. There is a variety of applications to both natural and social sciences.

I have included the following innovation for instructors who wish to introduce the transcendental functions early. At the end of Chapter 2 on derivatives, there is a section beginning an alternate track on transcendental functions, and each of Chapters 3 through 6 have alternate track problem sets on transcendental functions. This alternate track can be used to provide greater variety in the early problems, or can be skipped in order to reach the integral as soon as possible. In Chapters 7 and 8 the transcendental functions are developed anew at a more leisurely pace.

The book is written for average students. The problems preceded by a square box go somewhat beyond the examples worked out in the text and are intended for the more adventuresome.

I was originally led to write this book when it became clear that Robinson's infinitesimal calculus could be made available to college freshmen. The theory is simply presented; for example, Robinson's work used mathematical logic, but this book does not. I first used an early draft of this book in a one-semester course at the University of Wisconsin in 1969. In 1971 a two-semester experimental version was published. It has been used at several colleges and at Nicolet High School near Milwaukee, and was tested at five schools in a controlled experiment by Sister Kathleen Sullivan in 1972–1974. The results (in her 1974 Ph.D. thesis at the University of Wisconsin) show the viability of the infinitesimal approach and will be summarized in an article in the American Mathematical Monthly.

I am indebted to many colleagues and students who have given me encouragement and advice, and have carefully read and used various stages of the manuscript. Special thanks are due to Jon Barwise, University of Wisconsin; G. R. Blakley, Texas A & M University; Kenneth A. Bowen, Syracuse University; William P. Francis, Michigan Technological University; A. W. M. Glass, Bowling Green University; Peter Loeb, University of Illinois at Urbana; Eugene Madison and Keith Stroyan, University of Iowa; Mark Nadel, Notre Dame University; Sister Kathleen Sullivan, Barat College; and Frank Wattenberg, University of Massachusetts.

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